Formula for Quadratic Equation

The solutions of the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This formula should be memorised. The expression $(ax^2 + bx + c)$ may be made equal to any given quadratic expression, by giving a, b and c suitable values. To make $(ax^2 + bx + c)$ equal to $(5x^2 - 2x - 6)$, for example, put $a = 5$, $b = -2$, and $c = -6$.

The formula may be proved by the method of completing the square, applied to the equation $ax^2 + bx + c = 0$.

Rearrange:

$$ax^2 + bx = -c.$$

Divide through by a:

$$x^2 + \frac{b}{a}x = -\frac{c}{a}.$$

Add to each side ($\frac{1}{2}$ coefficient of x)^2:

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

or

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}.$$

Take the square root of both sides:

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}.$$

Subtract $\frac{b}{2a}$ from each side:

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

or

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$